Cosmos Overview

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ETR Aout 2017
Probabilistic Systems

Inherent Stochastic Systems

Telecommunication Protocols using Randomness

Chemical Reaction

Modelling of Unknown Parts of Systems

Waiting Queues

Banking System
Model Checking

Process algebra
Petri net
...

LTL
CTL
...

Transition system

$M \models \varphi$

$M \not\models \varphi$

Büchi automaton
Alternating automaton

$C \otimes A$
Model Checking for Stochastic Systems

Stochastic Process algebra
Stochastic Petri net
...

\[ p = \Pr(M \models \varphi) \]

DEDS

Deterministic hybrid automaton

\[ M \]

\[ \varphi \]

PCTL
CSL
...

\[ C \otimes A \]
Numerical vs Statistical Approaches

\[
\Pr \left( \begin{array}{cccc}
1 & 0.5 & 0.8 & 1 \\
0.5 & 2 & 0.4 & 0.2 & 0.3 & 3 & 0.3 & 5 & 1 \\
4 & 1 & & & & & & & \\
\end{array} \right) \models \neg \bullet U \bullet
\]
Numerical vs Statistical Approaches

Pr

\[
\begin{pmatrix}
\begin{array}{cccccc}
-1 & 0.5 & 0.5 & 0 & 0 & 0 \\
0 & -1 & 0.2 & 0.8 & 0 & 0 \\
0.4 & 0 & -1 & 0.3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{array}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\]

\[x_1 = 0.2368\]
Numerical vs Statistical Approaches

Pr

\[
\begin{pmatrix}
1 & 0.5 & 0.8 & 1 \\
0.5 & 0.4 & 0.2 & 0.3 & 1 \\
0.3 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\(\models \not\exists \cup \circ \)

Statistical Model Checking

- \(1 \rightarrow 2 \rightarrow 4 \times\)
- \(1 \rightarrow 3 \rightarrow 1 \rightarrow 3 \rightarrow 5 \checkmark\)
- \(1 \rightarrow 3 \rightarrow 4 \times\)
- \(1 \rightarrow 3 \rightarrow 1 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 4 \times\)
- \(1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \checkmark\)
- \(\ldots\)

\(x_1 \in [0.19, 0.26]\)
Numerical vs Statistical Approaches

Numerical Model Checking

\[
\begin{pmatrix}
-1 & 0.5 & 0.5 & 0 & 0 \\
0 & -1 & 0.2 & 0.8 & 0 \\
0.4 & 0 & -1 & 0.3 & 0.3 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
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x_3 \\
x_4 \\
x_5
\end{pmatrix} =
\begin{pmatrix}
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0 \\
0 \\
0 \\
1
\end{pmatrix}
\]

\[x_1 = 0.2368\]

Statistical Model Checking

1 \rightarrow 2 \rightarrow 4 \times \\
1 \rightarrow 3 \rightarrow 1 \rightarrow 3 \rightarrow 5 \checkmark \\
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\ldots
\]

\[x_1 \in [0.19, 0.26]\]
Numerical vs Statistical Approaches

Numerical approach

- Precise value (but prone to numerical errors)
- Strong probabilistic hypotheses
- Memory space proportional to the size of the stochastic process

Statistical approach

- Confidence interval: probabilistic framing
- Small memory space
- Easy to parallelise
- Weak probabilistic hypotheses (only an operational semantic)
- Requires fully stochastic models
- Rare events problem
**Numerical vs Statistical Approaches**

**Numerical approach**
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- Strong probabilistic hypotheses
- Memory space proportional to the size of the stochastic process

**Statistical approach**
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Discrete Event Dynamic System (DEDS)

DEDS: \( (S, S_0, E, \delta, (E_n)_0^\infty, (T_n)_0^\infty) \)

- A discrete set of state \( S \), initial state is a random variable (RV) \( S_0 \in S \)
- A set of events \( E \)
- A transition function \( \delta : S \times E \to S \)
- A sequence of RV \( (E_n)_0^\infty \). The sequence of states is \( S_{n+1} = \delta(S_n, E_n) \).
- A sequence of RV in \( \mathbb{R}^+ \): \( (T_n)_0^\infty \)

DEDS realisation example
Cosmos

DEDS modeled as a Stochastic Petri Net with general distribution

Hybrid Automaton Stochastic Logic = Linear Hybrid Automaton + Expressions

Synchronisation

- DEDS generates random timed words.
- Automaton tries to read the word.
- Expressions are evaluated on the variable of the automaton.
Generalised Stochastic Petri Net

Example Description (Tandem Queues)

- $Q_1$: Exponential(1.0)
- $Q_2$: Deterministic(1.0)
- Uniform(1,3)

A Petri net defines state space, events and transitions. After a transition is enabled, the time before firing is distributed according to the distribution. The next event is the transition with the smallest firing time.

Extensions:
- Petri net with inhibitor arcs, marking dependant valuation.
- Coloured Petri net.
Generalised Stochastic Petri Net

Example Description (Tandem Queues)

- Exponential(1.0)
- Deterministic(1.0)
- Uniform(1,3)

Q₁ → ⬠ Q₂

Description

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- After a transition is enabled the time before firing is distributed according to the distribution.
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Generalised Stochastic Petri Net

Example Description (Tandem Queues)

Exponential(1.0)  Deterministic(1.0)  Uniform(1,3)

Q1  Q2

Description

- A Petri net; defines state space, events and transitions.
- After a transition is enabled the time before firing is distributed according to the distribution.
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Extensions

- Petri net with inhibitor arcs, marking dependant valuation.
- Coloured Petri net
Petri Net Demo
System Example

Flexible Manufacturing Systems

```
real alpha1=0.45;
real alpha2=0.55;
real Tunit=0.8;
real mu1 = 0.714285714285714302;
real mu2 = 0.367142857142857151;
real sigma = 0.714285714285714302;
int l1=3;
int l2=3;
```

Arrive \text{UNIFORM}(\alpha_1,\alpha_2)

In1,2 \quad \text{DETERMINISTIC}(0)

In1,3 \quad \text{DETERMINISTIC}(\text{Tunit})

Products

InConv1

In1,1

InConv2

In2,2

In2,3

In2,1

LOGNORMAL(\mu_1,\sigma_1)

Serve1

LOGNORMAL(\mu_2,\sigma_2)

Serve2
System Example

Molecular Signalling Pathway

RasGTP

Raf_RasGTP

Phase 1

RafP

Phase 2

MEK_RafP

MEKPP_RafP

Phase 3

ERK_MEKPP

ERKPP
System Example

Human Heart and Pacemaker system
System Example
Computation with DNA
Given a set of trajectories obtained by simulation, what can we compute?

\[ S_1 \xrightarrow{T_1,E_1} S_2 \xrightarrow{T_2,E_2} S_3 \xrightarrow{T_3,E_3} S_4 \xrightarrow{T_4,E_4} \ldots \xrightarrow{T_{n-1},E_{n-1}} S_n \]
Given a set of trajectories obtained by simulation, what can we compute?

\[ s_1 \xrightarrow{T_1,E_1} s_2 \xrightarrow{T_2,E_2} s_3 \xrightarrow{T_3,E_3} s_4 \xrightarrow{T_4,E_4} \ldots \xrightarrow{T_{n-1},E_{n-1}} s_n \]

**Linear Hybrid Automaton (LHA)**

- An automaton labelled by set of DEDS events or \(\#\).
- A set of variables with flows.
- Assignment of variable.
- Linear guard and invariant.
LHA Semantic

Two kinds of transitions

- Synchronised transition
  ⇒ DEDS and LHA change state at the same time

- Autonomous transition (#)
  ⇒ only the LHA changes location, as soon as the guard is satisfied
LHA Semantic

Two kinds of transitions

- Synchronised transition ⇒ DEDS and LHA change state at the same time
- Autonomous transition (♯) ⇒ only the LHA changes location, as soon as the guard is satisfied

Time behaviours

- Flows of clocks are expressions on the state of the DEDS ⇒ Piece-wise linear
- Guards are linear expressions on variables ⇒ guard satisfaction boils down to solving linear system
LHA Semantic

Two kinds of transitions

- Synchronised transition
  \( \Rightarrow \) DEDS and LHA change state at the same time

- Autonomous transition \((\#)\)
  \( \Rightarrow \) only the LHA changes location, as soon as the guard is satisfied

Time behaviours

- Flows of clocks are expressions on the state of the DEDS
  \( \Rightarrow \) Piece-wise linear

- Guards are linear expressions on variables
  \( \Rightarrow \) guard satisfaction boils down to solving linear system

Determined

- One initial location
- Final locations
- The automaton is deterministic
Hybrid Automata Stochastic Logic (HASL)

HASL formula

- An LHA
- An expression over variables of the automaton, to compute complex indexes on the *accepted* path.
Hybrid Automata Stochastic Logic (HASL)

**HASL formula**
- An LHA
- An expression over variables of the automaton, to compute complex indexes on the *accepted* path.

**Formula construction**

```
AVG ( Integral( t - m ) + Last( m ) )
```

- **AVG**: Probabilistic operator
- **Integral**: Path operator
- **Last**: Path operator
- **( t - m )**: Linear expression
Hybrid Automata Stochastic Logic (HASL)

HASL formula

- An LHA
- An expression over variables of the automaton, to compute complex indexes on the accepted path.

Formula construction

\[
\text{AVG} \left( \begin{array}{c}
\text{Integral} ( t - m ) \\
\text{Last} ( m ) 
\end{array} \right)
\]

Probabilistic operator

- \text{PROB}
- \text{AVG}(X)
- \text{PDF}(X, \text{step}, \text{min}, \text{max})
- \text{CDF}(X, \text{step}, \text{min}, \text{max})

Path operator

- \text{Last}( x )
- \text{Integral}( x )
- \text{Mean}( x )
- \text{Min}( x ) / \text{Max}( x )
Hasl evaluation

Synchronisation

- Simulation of the GSPN
- Synchronisation of the LHA
- Trajectory is accepted if a final state is reached
- Trajectory is rejected if LHA fail to synchronise
### Hasl evaluation

#### Synchronisation
- Simulation of the GSPN
- Synchronisation of the LHA
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#### Hasl expression
- Linear expression evaluated after each step of simulation
- Path expression evaluated along the path
- Probabilistic operator evaluated on set of trajectories
Hasl evaluation

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Hasl expression

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Trajectories are not stored!

No dynamic allocation of memory!
Confidence Interval

Given a random variable $X$ and a confidence level $1 - \varepsilon$, an estimator of the expected value of $X$ returns a confidence interval $I$ if

$$\Pr(E(X) \in I) \geq 1 - \varepsilon$$

Three parameters: confidence level, confidence interval width and number of samples. Two of them have to be fixed.
Cosmos 1/2

Description: a command-line tool

- Input model: a Generalised Stochastic Petri Net
- Input specification: HASL formulas
- Input: Statistical Parameters
- Output: Probabilistic framing of values of HASL formulas

Architecture

- Contains 25 Kloc of C/C++ and OCaml under GPLv2
- Generates code implementing the synchronisation GSPN/LHA
- Distributes simulation

GSPN
LHA
Code Generator

GSPN.cpp
LHA.cpp
CC

Cosmos runtime
Executable
Statistical Engine
Simulator

Cosmos results
runs
Cosmos 1/2

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- **Generates code implementing the synchronisation GSPN/LHA**
- **Distributes simulation**

![Diagram of Cosmos architecture](image)
Cosmos 2/2

Features

- Static and Sequential statistical methods: Chernoff-Hoeffding, Chow-Robbins, Gaussian, SPRT
- Several input formats: GrML, Marcie, PNML, Prism
- Several compatible editing tools: Coloane, GreatSPN Editor, Snoopy
- Plain and coloured Petri nets
- Fast thanks to structural analysis of Petri net and code generation
- Low memory footprint
- Various possible outputs

Extensions

- Handling of Rare Events with importance sampling
- Uniform sampling for time automata
- Hardware in the loop simulation
- Simulation of hybrid models: Simulink
Cosmos 2/2

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Conclusion

- Fast and lightweight statistical model checker.
- Rich classes of input models.
- Rich specification language
- Modular and open source → easy to hack

Download: http://www.lsv.ens-cachan.fr/Software/cosmos/
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